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# A BAYESIAN APPROACH TO PASSIVE BEARINGS ONLY TARGET TRACKING

BY R. S. HEBBERT L. T. BARKAKATI

UNDERWATER SYSTEMS DEPARTMENT

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# **NAVAL SURFACE WARFARE CENTER**

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# FOREWORD

A major objective of digital processing of sonobuoy signals is to generate target tracks. This report describes a passive tracker based on a scheme that updates the target probability density function over a rectangular grid. The theory and results presented here would be beneficial to other researchers working on passive trackers.

Approved by:

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C. A. Kahvitinos

Sensors and Electronics Division

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#### SECTION 1

#### INTRODUCTION

Passive directional sonobuoys (DIFAR's) can provide estimates of the bearing, the standard deviation of the bearing, and the signal strength of a sound source (the "target"). The estimates from three or more buoys will not produce an exact target location (see Figure 1). Rather, they will only indicate a region where the target is likely to be present. It is necessary to have some method of reconciling multiple estimates of target bearings (from different buoys) to locate a target.

In typical passive tracking algorithms, the target tracker is an extended Kalman filter with the usual assumption of Gaussian covariances. This conventional tracker is computationally effective but not very robust for maneuvering targets. Also, it is difficult to initialize the Kalman filter. These limitations of the conventional tracker have prompted us to examine a passive tracker using probability density functions (pdf's). In the digitial implementation of such a tracker, one assigns probabilities of the target being present at each point in a grid (see Figure 1). The target location estimate is obtained by maximizing this probability, i.e., it is a maximum-a-posteriori (MAP) estimate (see Appendix A for details). The trackers using pdf's vary in their assumptions of prior knowledge about target motion. The tracker described in this report assumes no knowledge about target motion except that its speed be bounded above. If a velocity estimate was available, the predicted probability density would take advantage of this.

The pdf tracker can be very computation intensive because the probabilities at all the grid points have to be updated at eact time step. In the proposed method, we have strived to lessen the computational load by

updating the probabilities at only those grid points lying within the sector  $\phi_i \pm \delta_i$  for the i-th buoy, where  $\phi_i$  is the bearing estimate produced by the i-th buoy and  $\delta_i$  is a quantity that depends on the standard deviation of the estimate (see Figure 1). The updating of the pdf at grid points is described in Appendix B.

One way to generate target tracks using pdf's is to produce MAP estimates of the target location at each time step, using bearing estimates for that step alone. In this method one recomputes the probabilities at all grid points at each time step. In our approach, we retain all probabilities and propagate them between time steps, i.e., the probabilities at the grid points around the current estimated target position are updated before starting the processing for the next time step.

The proposed method achieves further computational efficiency by using the natural logarithm of the pdf's at each grid point. Because of this choice, the updating of the log probabilities at successive time steps involves simple additions rather than multiplications.

In the next section, we describe the proposed tracker. In Section 3, we present the results obtained by this tracker on simulated as well as real data.

#### SECTION 2

#### PASSIVE PDF TRACKER

Consider an NxN grid in the x-y plane (see Figure 2) with M sonobuoys and a single target track. The data from the i-th buoy provides target bearing estimate,  $\phi_i$ ; signal-to-noise ratio (SNR),  $\phi_i$ ; hence a measure of standard deviation,  $\sigma_i$ . Note that the bearing angles are measured clockwise from the x-axis as shown in Figure 2.

Suppose the bearing of the (i,j)-th grid point is  $\phi_{ij}^k$  with respect to the k-th buoy. The model for the conditional pdf of the actual target being  $\phi_{ij}^k$  given the bearing estimates  $\overline{\phi}_k$  is assumed to be:

$$P(\phi_{i,j}^{k} | \overline{\phi}_{k}) = c e^{\rho_{k}} \cos(\phi_{i,j}^{k} - \overline{\phi}_{k})$$
 (1)

where C is a normalizing constant.

This model has several attractive features. The cosine function makes it symmetric and periodic about  $\overline{\phi}_k$ . The probability density narrows with increasing SNR. The exponential nature allows successive updating through additions, provided we look at  $\ln p(\phi_{ij}^k/\overline{\phi_k})$  rather than Equation (1).

The tracker is initialized by setting the log probability to zero at all grid points (we are neglecting the normalizing constants). This amounts to assuming a uniform pdf.

At each time step and for each buoy, one updates the probabilities at all grid points lying within the sector  $\phi_i \pm \delta_i$  degrees. This involves evaluating the exponent of Equation (1) and adding the computed value to the previous log probabilities at a given grid point.

After doing this for all M buoys, the target location is estimated by searching for the maximum of the probabilities at all grid points.

The next step, prediction, involves "blurring" or propagating the pdf to account for target motion. Since we assume no knowledge of the direction of motion of the target, we diffuse the probabilities symmetrically at all grid points (see Appendix C for details).

The process is then repeated with new observation for the next time step.

#### SECTION 3

#### RESULTS

#### SIMULATED DATA

The simulated data shown was generated for a target executing a sharp turn. Two cases are presented: in the first there was no observation noise in the bearing estimates; in the second case there was observation noise. The data were generated for three buoys.

The tracker was implemented on VAX-11/780 computer. A 70x70 grid was used. The actual simulated track and the positions determined by the tracker are shown in Figures 3 and 4 for the first and second case respectively.

# REAL DATA

The measurements of a real track were used next. There are inherent limitations to successful processing of real data. One major problem is the uncertainty of the sonobuoy position. Another limiting factor is the error in the bearing estimates provided by the buoys. A glaring example of these real-life problems is the inconsistency in the results shown in Figures 5 and 6. The true track of the target along with the locations of the four buoys are shown in Figure 7. Note that the track estimated by using data from all four buoys (Figure 6) is much worse than that obtained in the absence of the first buoy from the left. One possible explanation of this inconsistency is that the first buoy is malfunctioning or its position estimate is in gross error.

We have made some preliminary attempts to overcome the above-mentioned limitations of real data. The first approach is to move each buoy around its nominal position so as to minimize the residual error in the bearing estimates computed from the MAP estimate of the target location. This approach was first applied to the data from the three "good" buoys and the result is shown in Figure 8. This track is very close to the one obtained without any error minimization scheme (Figure 5). When the method is applied to the data from the four buoys, the first buoy has to be placed quite far (see Figure 9) from its given location to reduce the error. These results suggest that the first buoy position may be erroneous. The improvement in the result (Figure 9) with this method over that without (Figure 6) indicates that this method may be useful in improving the quality of track estimates from real data.

The effect of biased-bearing estimates may be minimized by artificially increasing the variance of the estimates so that in effect we attach less confidence to these quantities. This was also done with the data from three and four buoys. The resulting tracks are shown in Figures 10 and 11. With the three good buoys, the track is hardly changed. When the fourth, bad, buoy is included, the track estimate is improved.

#### SECTION 4

# DISCUSSIONS AND FUTURE DIRECTIONS

The current version of the proposed tracker does not assume any estimate of the target speed. Only a limit on the target's speed is imposed. Because of this, the probabilities were "blurred" so as to account for the maximum possible speed. It is anticipated that the availability of an estimate of the target's speed will result in a better track estimate.

This tracker could be implemented using time delay measurements as well. This is because position estimates can be obtained from time delays. Also note that this technique could be used to track multiple targets, especially if the sensor had multiple bearings or time delays.

Another topic of future research is the study of the effect of the grid pattern on the performance of the tracker. The implemented version of the tracker uses a rectangular grid. A likely candidate for the grid in future implementations is a hexagonal one.

Doppler shift measurements provide the derivative of position and requires different treatment. A possible doppler/bearing pdf tracker could be implemented by the following steps:

- Step 1: Search X and Y coordinates for target location bearing estimates.
- Step 2: At the target location compute target velocities X, Y, using doppler shifts.

Step 3: Use the velocity estimates to appropriately predict the probabilities for the next time cut.

The basic idea of this pdf tracker is sound as indicated by the results from the simulated data. Further work is necessary to overcome the above-mentioned practical problems inherent in processing real data.

Lastly, this pdf tracker has been implemented in a computationally efficient manner. It is anticipated that distributed array processors would help lower the computational load. Further work is needed to investigate this.

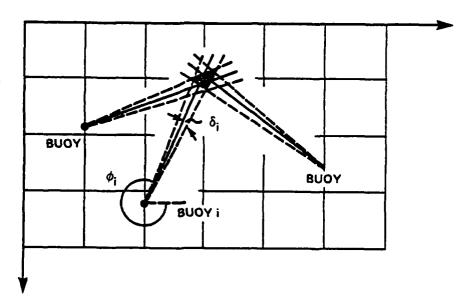


FIGURE 1. UNCERTAIN TARGET LOCATION FROM THREE BUOY MEASUREMENTS

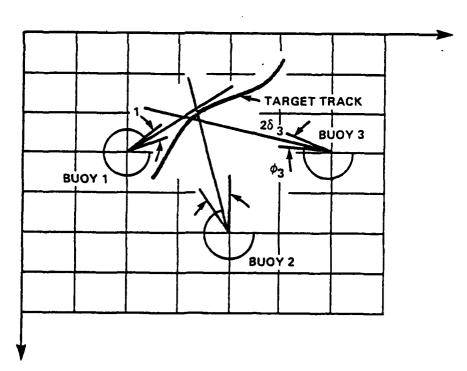


FIGURE 2. MULTIPLE BEARING ESTIMATES FOR A SINGLE TARGET TRACK

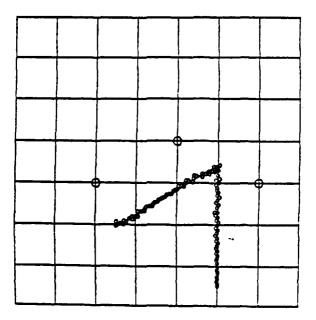


FIGURE 3. THE ACTUAL SIMULATED TRACK AND THE OUTPUT OF PDF TRACKER WITHOUT OBSERVATION NOISE

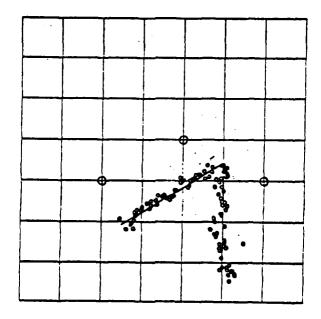


FIGURE 4. THE ACTUAL SIMULATED TRACK AND THE OUTPUT OF PDF TRACKER WITH OBSERVATION NOISE

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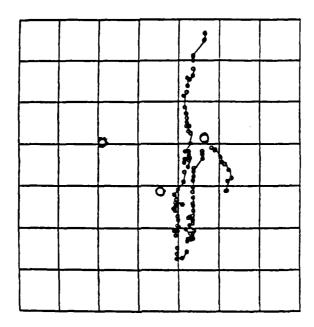


FIGURE 5. THE OUTPUT OF PDF TRACKER WHEN USING REAL DATA FROM THREE BUOYS

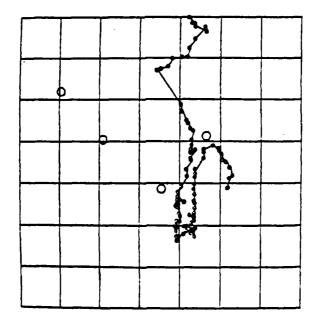


FIGURE 6. THE OUTPUT OF PDF TRACKER WHEN USING REAL DATA FROM FOUR BUOYS

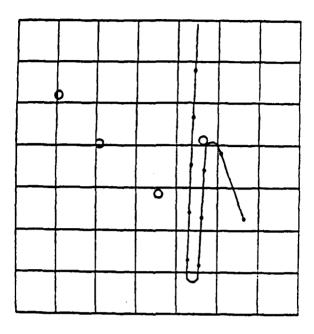


FIGURE 7. THE TRUE TRACK OF REAL DATA

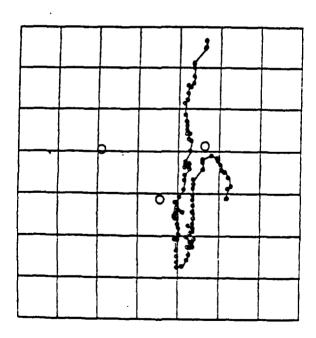


FIGURE 8. THE OUTPUT OF PDF TRACKER WHEN USING REAL DATA FROM THREE MOVING BUOYS

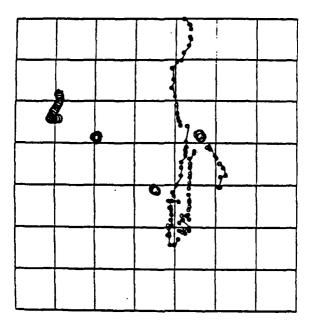


FIGURE 9. THE OUTPUT OF PDF TRACKER WHEN USING REAL DATA FROM FOUR MOVING BUOYS

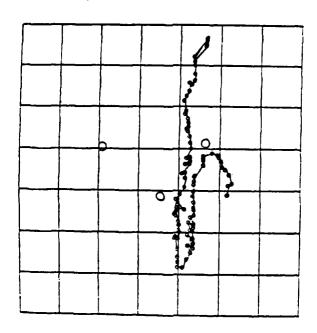


FIGURE 10. THE OUTPUT OF PDF TRACKER WHEN USING REAL DATA FROM THREE BUOYS WITH ADDED VARIANCE

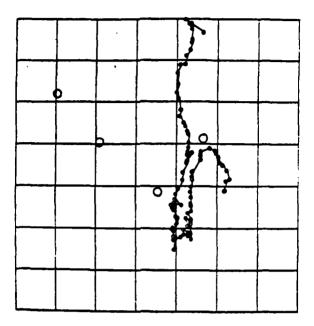


FIGURE 11. THE OUTPUT OF PDF TRACKER WHEN USING REAL DATA FROM FOUR BUOYS WITH ADDED VARIANCE

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#### APPENDIX A

#### APPLICATION OF BAYES RULE IN PDF TRACKER

Let x and y denote random variables, possibly multidimensional, having the probability density functions (pdf) p(x) and p(y) respectively. The joint density function is denoted by p(x,y), while p(x/y) and p(y/x) are the conditional densities. Then the Bayes rule for probability density functions is:

$$p(x,y)=p(x/y)p(y)=p(y/x)p(x)$$
(A-1)

or, ignoring normalization: p(x/y)=p(y/x).p(x)

In the pdf tracker, x represents the target coordinates while y denotes the bearing estimates (or the time delay estimates). Then p(x) denotes the prior pdf of the target being present at location x, p(x/y) is the "a posteriori" pdf of the target location given the observation y.

The maximum a-posteriori (MAP) estimate,  $\mathbf{x}_{\text{MAP}}$  is given by

$$x_{MAP} = \max_{x} p(x/y) \tag{A-2}$$

This pdf tracker determines  $x_{MAP}$ . It proceeds with an assumed p(y/x). The assumed form of p(y/x) is exponential and since Equation (A-2) is equivalent to

$$x_{MAP} = Max \ln p(x/y)$$
 (A-3)

The computations are greatly facilitated by using the log pdf's. Using Equation (A-1) we can write Equation (A-3) as

$$\mathbf{x}_{MAP} = \max_{\mathbf{x}} \left[ \ln p(\mathbf{y}/\mathbf{x}) + \ln p(\mathbf{x}) \right] \tag{A-4}$$

which reduces the multiplications to cheaper additions. By taking the logarithm, we also avoid evaluating a transcendental function in p(y/x).

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#### APPENDIX B

# UPDATING THE PDF IN THE GRID

The pdf tracker described in this report updates the pdf in a sector rather than at all points in the grid. Suppose y denotes the bearing observations (or time delays) for a target whose coordinates are denoted by x. Then the pdf p(y/x) should be maximum at the observed bearing, y<sub>0</sub>, and decay as we move away from y<sub>0</sub>. In this tracker, we have assumed that ln p(y/x) is zero at all points outside the sector y<sub>0</sub>  $\pm$   $\delta$ <sub>0</sub> where  $\delta$ <sub>0</sub> is a quantity that depends on the signal-to-noise ratio (SNR) at the buoy which provided the estimate y<sub>0</sub>. We pick  $\delta$ <sub>0</sub> so that the larger the SNR, the smaller the sector size.

Our choice for the sector has several advantages. The most prominent one is that we substantially reduce the computational burden of the tracking algorithm. The computational load with our updating scheme is an order of magnitude less than what would be necessary for an "all points" upating scheme.

Another important advantage is that if an occasional bad observation occurs, it will not affect the pdf in the vicinity of the true target location. Thus the bad datum will not bias the target pdf.

#### APPENDIX C

#### BLURRING THE PDF

The pdf tracker described in this report makes no assumption about the target velocity except that it imposes an upper bound on the speed (velocity has direction, speed doesn't). The bearing measurements are given some time apart, e.g., 15 seconds. If we assume a speed limit of 9 yds/sec(- 15 knots), then the target could be anywhere within a circle of radius 135 yds (assumed 15 second interval of measurements by the next time cut. It is therefore necessary to update the prior target pdf's before processing the data for the next time cut. This updating process is called "blurring", i.e., the pdf at each grid point is in effect spread out uniformly over a disk of radius R (e.g., 135 yd in the above example). The discrete structure of the grids does not allow the disk to be completely circular, but an octagonal shape is achieved through the scheme shown in Figure C-1.

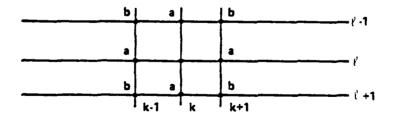


FIGURE C-1. GRID POINTS FOR BLURRING

In Equation (C-1), let D1 be the maximum of the pdf evaluated at the points labeled "a" (see Figure C-1). Similarly, suppose D2 is the maximum among the points denoted "b". Then the updating is done through the FORTRAN statement

$$P(k,1)=(1-E1-E2)*P(k,1)+(E1*D1+E2*D2)$$
 (C-1)

The left-hand side of the above equation is the prior pdf of the current time and the right hand side is the "a-posteriori" density of previous time

$$0 < E1 < 0.6$$
 (C-2)

and

### E2/E1-2/3

The choice of E2 was made through experimentation. The condition (Equation (C-2)) is necessary to ensure stability of the filter (Equation (C-1)). In Equation (C-1) the value of El determines the amount of blurring pattern.

If the problem parameters (speed, grid size, and time between observations) are such that the above blurring formula is not sufficient, then Equation (C-1) may be applied more than once to obtain the correct amount of "blurring".

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